ECE 580 Project 1

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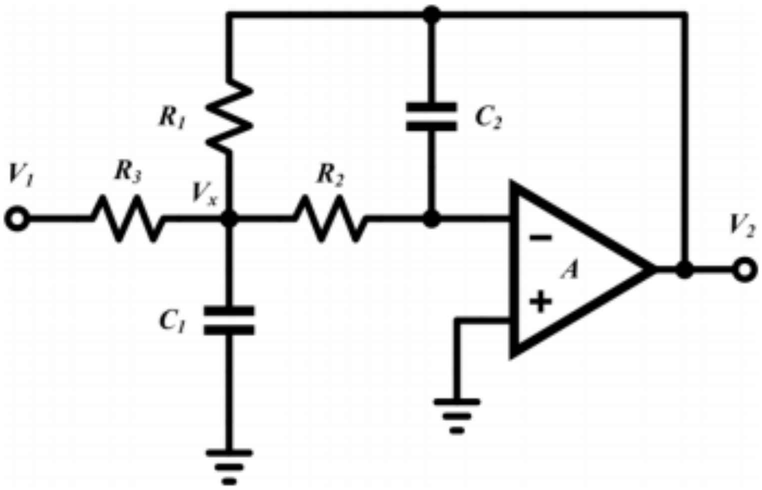


Figure 1 - Rauch Filter Circuit

Specifications:

- Pass-band 0 – 20 kHz;
- DC gain = 1;
- 3-dB bandwidth 100 kHz;
- Equal-valued capacitors;
- Maximum thermal pass-band noise 1 μV

Requirements
1. Find R2/R1 so that Q is maximized. What is Qmax?
2. Find all resistances for minimum power dissipation.
3. Find the capacitances.

Abstract steps taken in chronological format
1.Find parameters required to achieve Q max and DC Gain of 1
2.Find resistances based off of noise specification of $1uV^2$
3.Find capacitances based off of 3dB frequency

Step 1: Extracting Parameters using Q max

It is given that the transfer function for a second order low-pass filter is as follows.

$$H(s) = \frac{A_v \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

The transfer function of the Rauch filter is given below.

$$H(s) = \frac{\frac{G_3}{G_1} \frac{G_1 G_2}{C_1 C_2}}{s^2 + \frac{G_1 + G_2 + G_3}{C_1} s + \frac{G_1 G_2}{C_1 C_2}}$$

By substituting expressions we can determine that,

$$A_v = \frac{G_3}{G_1}$$

$$\omega_0^2 = \frac{G_1 G_2}{C_1 C_2}$$

$$\frac{\omega_0}{Q} = \frac{G_1 + G_2 + G_3}{C_1}$$

Also, the filter will have two real poles or a pair of complex conjugate poles depending on the Q. Either way ω_0 will be the geometric mean of the poles.

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

To achieve a DC gain of 1 we set $G_1 = G_3 = G$ and since both capacitors have been specified to be equal we set $C_1 = C_2 = C$. Also, from the hand out $Q = \frac{R_2}{R_1} = \frac{G_1}{G_2} = \alpha > 0$. Plugging these identities into the Rauch filter transfer function we obtain the following,

$$H(s) = \frac{\frac{1}{\alpha}\frac{G^2}{C^2}}{s^2 + \frac{G\left(2+\frac{1}{\alpha}\right)}{C}s + \frac{1}{\alpha}\frac{G^2}{C^2}}$$

From the simplified transfer function we can derive that,

$$Q(\alpha) = \frac{\sqrt{\alpha}}{2\alpha + 1}$$

and that Q_{max} occurs when $\alpha = \frac{1}{2}$ where $Q\left(\frac{1}{2}\right) = \frac{1}{2\sqrt{2}}$. Since $Q_{max} < \frac{1}{2}$ the poles are real and will be,

$$\omega_1 = \frac{G}{C} = \frac{1}{R_1 C_1}$$

$$\omega_2 = \frac{1}{\alpha}\frac{G}{C} = \frac{1}{R_2 C_2}$$

Finally, to satisfy the 3dB corner frequency specification we will set $f_{3dB} = 100kHz$.

Step 2: Extracting R-Value using Noise Requirement

Because the 3dB frequency is much larger than the pass-band limit of 20kHz the thermal noise due to the resistors can be calculated by the three following equations:

$$V_1^2 = 4KTR * 1^2 * \Delta f$$
$$V_2^2 = 4KTR * \left(\frac{R1}{R3}\right)^2 * \Delta f$$
$$V_3^2 = 4KTR * \left(\frac{R1 + R3}{R3}\right)^2 * \Delta f$$

Where V1, V2, V3 is the noise due to R1, R2, R3 respectively and $\Delta f = 20kHz$.

Total noise is then:

$$V_1^2 + V_2^2 + V_3^2 = V_{total}^2 = 1uV^2$$

Solving for R, where R=R1 we find R is 773Ω and from the previous sections, we know that R1=R3, R2=R1/2.

R1=773Ω
R2=386Ω
R3=773Ω

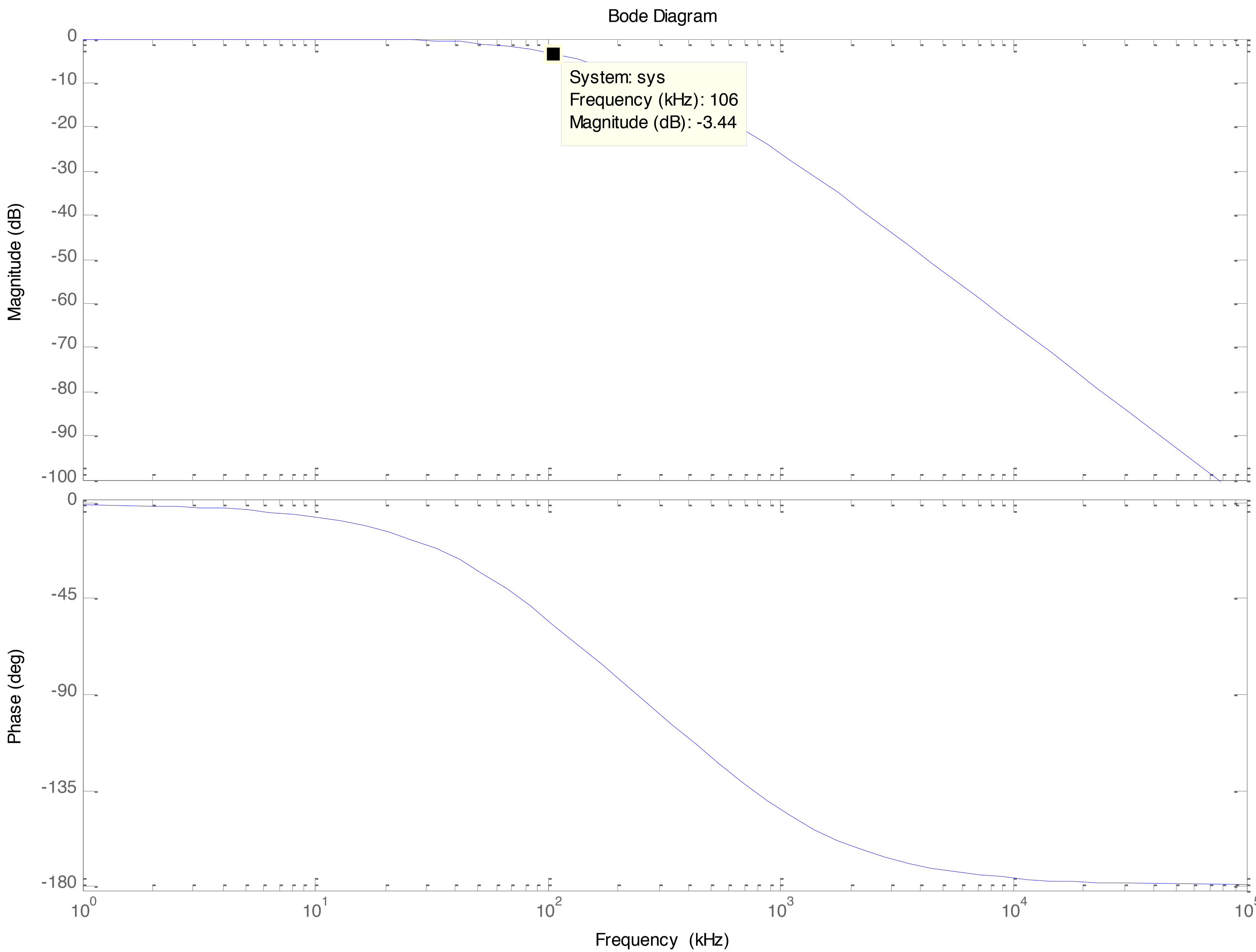
Step 3: Extracting Capacitor Value using 3dB frequency

$$|H(j\omega_{3dB})| = \frac{1}{|1 - \left(\frac{\omega_{3dB}}{\omega_0}\right)^2 + \frac{j\omega_{3dB}}{Q\omega_0}|} = \frac{1}{\sqrt{2}}$$

Solving for C when $R = 773\Omega$ and $f_{3dB} = 100$kHz yields $C = C_1 = C_2 = 1.21nF$.

Results

Parameter	Value
Q_{max}	$\frac{1}{2\sqrt{2}}$
R_2/R_1	$\frac{1}{2}$
R_1	773Ω
R_2	386Ω
R_3	773Ω
C_1	$1.21nF$
C_2	$1.21nF$


Bode Diagram
System: sys
Frequency (kHz): 106
Magnitude (dB): -3.44
Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
Phase (deg)
0
-45
-90
-135
-180
10^0
10^1
10^2
10^3
10^4
10^5
Frequency (kHz)